Proliferation and Entry Deterrence in Vertically Differentiated Markets

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Abstract

We examine the profitability, entry deterrence and welfare effects of proliferation offered by non-cooperative firms competing in quality and price. In a market of one high quality firm and competitive low quality firms, we find that the established high quality firm will not initiate proliferation but may have an incentive to do so if facing entry threats. The proliferation quality is endogenously determined and the industry profits decrease with such proliferation. Moreover, we show that proliferation increases consumer surplus in the same way as entry does. That is, while proliferation to deter entry is anti-competitive, it is not necessarily welfare-reducing.

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1 Introduction

In vertically differentiated markets with products of the same generic type, firms making quality-price decisions face a trade-off between softening competition and broadening market segments. As a result, proliferation or multi-product strategy (see, e.g., Constantatos and Perrakis, 1997), with which firms typically offer a range of products, could have either procompetitive or anticompetitive effects. On the one hand, by filling in the gaps on a quality spectrum, proliferation increases competition as products available become less differentiated. On the other hand, proliferation enables firms to reach different market segments and better discriminate against consumers.

Ever since Schmalensee (1978) suggests empirically that an incumbent can preempt entry by introducing new products and restricting the market space available to potential entrants, there has been increasing antitrust concerns against excessive proliferation. If one tries to rationalize firms' engagement in proliferation that is not *per se* profitable, the argument might be that it is optimal if some other considerations are taken into account, e.g., entry deterrence.

Proliferation to deter entry has been under intense investigation in horizontal differentiation settings (Judd, 1985; Choi and Scarpa, 1991; Murooka, 2012), but the same analysis in vertical settings is sparse. Given that excessive proliferation is suspicious, one may tempt to assume that moderate proliferation is more likely to be justified on profitability ground thus seemingly causes no harm. In this paper, we show that it is possible for vertical proliferation to be completely undesirable in the absence of entry threats, even in the extreme case of introducing only one additional quality. Therefore proliferation of any level might involve anticompetitive purpose. More interestingly, when proliferation is optimal given entry threats, it always benefits consumers.

In a vertically differentiated market, suppose that consumers have identical ordinal preferences over product quality and differ only in income levels.[1] Then at an equal price,

[1] Another interpretation of vertical differentiation is by Mussa and Rosen (1978) where consumers differ only in their intensity of preference for quality. The two interpretations are related; if θ is a parameter of intensity of

higher quality is preferable. For example, people would generally agree that a Mason Pearson hairbrush is preferable to a hairbrush from Primark.[2] In a simple duopoly market, a series of papers (Gabszewicz and Thisse, 1979; Shaked and Sutton, 1982; Donnenfeld and Weber, 1992) prove the existence of a unique quality-price equilibrium in which two established firms follow the *Principle of Maximal Differentiation*. That is, they first choose distinct qualities and then set distinct prices, so as to dampen competition and increase profits.

It is also common for a single firm to offer quality-differentiated products to consumers. For example, for a given size, Mason Pearson usually offers three types of hairbrush, namely pure bristle, bristle/nylon, and nylon, where bristle is finer and more expensive than nylon as the component of a hairbrush head. Another form of proliferation, namely brands collaboration, is usually conducted jointly by two firms. Similar to a joint venture strategy, brands collaboration is defined by Chun and Niehm (2010) as a strategic and cooperative relationship where brands devote their own competitive advantages to present products under joint names to consumers.

A good example of vertical brands collaboration is the high-street brands (e.g. Gap and H&M) and luxury brands (e.g. Jimmy Choo and Lanvin) collaborations in the fashion apparel and sportswear industry. The premier case was by Puma and Jil Sander in the late 1990s. Chun and Niehm (2010) suggest that through this collaboration, "the boundaries between genres collapsed." In 2003, Adidas launched its first collaborations with designer brands Stella McCartney and Jeremy Scott. Puma collaborated with Neil Barrett and Alexander McQueen in subsequent years. Nike joined the trend in 2012 and has been collaborating with Liberty.

A more typical case has been the Sweden based clothing retailer Hennes & Mauritz (H&M). In 2004, H&M tied up with Karl Lagerfeld and produced a successful collection. Since then, it has launched 21 collaborations with 16 different designer brands.[3] The average

preference for quality, then wealthier consumers have a higher θ and enjoy quality improvement more that less wealthy consumers (See, e.g., Tirole, 1988; Donnenfeld and Weber, 1992).

[2] See http://masonpearson.co.uk/ and http://www.primark.com/en/search/categorised search?q=hairbrush.

[3] Collaborations in the last decade included those with Stella McCartney, Viktor & Rolf, Roberto Cavalli, Comme des Garcons, Marimekko, Matthew Williamson, Jimmy Choo, Sonia Rykiel, Lanvin, Versace, Marni, Anna Dello

prices of items from H&M collaborations were over £100, with the most expensive collaboration so far being Maison Martin Margiela for H&M (on average £141.61 per item). Despite at much higher prices compared to normal H&M ranges, the collaboration ranges were sold out quickly. Some popular pieces were sold on eBay for five times the original prices. Meanwhile, the prices were lower than the original luxury collections. As reflected by prices, the perceived quality of collaborated ranges seems to be higher than the normal ranges from the high-street stores, although would not be as high as the original designer collections. As a result, quality configuration expands with such joint proliferation.

It is of importance to study proliferation in vertically differentiated markets since the existing studies relating to it is limited and does not keep up pace with the boom of the real world (joint) proliferation cases. Among many strategic incentives, proliferation has mainly been examined for two – to increase profit and to deter entry. While there is a literature on the optimality of proliferation (e.g., Shaked and Sutton, 1982), the specific conditions and endogenous proliferation qualities have been largely unaddressed, leaving it difficult to evaluate proliferation even in a simple framework. Since the entry deterrence effect of proliferation in vertical differentiation settings is rarely examined, the relevant welfare effects remain unclear.[4]

Although fully replicating the real world high-street and luxury collaborations is beyond the scope of this paper, we wish to examine how proliferation works in a context that captures this sort of competition. In the absence of entry threats, common expectation of firms on proliferation is to broaden market segments. Whether such expectation can be fulfilled is unclear without knowing how firms wish to locate the additional qualities.

Furthermore, when a firm makes decisions regarding proliferation, it faces a menu of options of how much control it wishes to take. For example, it may wish to take full control over the quality-price decision of any additional product introduced; or any additional product

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Russo, Maison Martin Margiela, Isabel Marant and Alexander Wang. See http://about.hm.com/en/About/facts-about-hm/people-and-history/history.html.

[4] There is a much richer strand of literature on "limit quality" to deter entry (e.g., Lane, 1980; Donnenfeld and Weber, 1995; Noh and Moschini, 2006) than proliferation to deter entry in vertical differentiation model.

acts as an independent unit that decides its own quality and price, while the parent-firm acts as a shareholder collecting profit from it (i.e., *divisionalisation* in Baye, Crocker and Ju, 1996). Immediately the message is that an entrant would make the same quality-price decision as an independent unit, as the independent unit seeks to maximise its own profit instead of the total profits of its parent-firm.

In a model of maximal differentiation where one firm specializes at the high end of the quality spectrum and Bertrand competitive firms operate at the low end, we show that the high quality firm would only be able to locate a middle quality product if it allows that product to be an independent unit. The quality level of the middle quality product is endogenously determined and is a convex combination of the existing high and low qualities. To be specific, it is 4/7 of that of the high quality product plus 3/7 of that of the low quality product. The price of the middle quality product is 2/7 of the price of the high quality product.[5] Such proliferation is not optimal in the absence of entry threats but becomes optimal in the presence of entry threats, when the proliferation cost is sufficiently small.

Moving from the two-quality to the three-quality market, if the fixed cost of introducing the third quality is small enough, then the rise in consumer surplus would outweigh the fall in industry profits, hence total welfare increases. Such increase in welfare could be achieved by the high quality firm through proliferation, or by the entrant if proliferation does not take place. From the welfare point of view, whether the outcome is achieved more efficiently by the established firm or by the entrant depends on their relative cost of introducing the third product. If the high quality firm benefits from the fact that it is already established hence incur lower cost, then not only proliferation can increase welfare, it also does in a way that is more efficient than entry.

The setting considered in this paper is along the similar lines to Tirole (1988), who provides an intuitive yet simple way to solve the model of vertical differentiation. We allow

[5] This is superficially similar to Choi and Shin (1992)'s results in a duopoly model of vertical differentiation with uncovered market. They find that the quality level of the low quality product is 4/7 of that of the high quality product, and the price of the low quality product is 2/7 of the price of the high quality product. However, in this paper, we have an additional middle quality product.

competitive firms in the low quality segment, which is different from the standard duopoly model. This modification, however, is consistent with the distribution of differentiated brands in the real world. For example, in the fashion apparel industry, there are countless high-street brands and far fewer designer luxury brands. Although it is a simplified model, it delivers a comprehensive evaluation of the proliferation strategy.

This paper is close in spirit to Baye *et al*. (1996), who study firms' incentives to add divisions before engaging in Cournot competition. They suggest that the profit generated by a new division may be offset by the fall in the profit of the parent-firm's existing units. Therefore proliferation, however moderate, might fail to fulfil the expectation.

The idea on proliferation location is close to Donnenfeld and Weber (1992), who show in a model of maximal differentiation that a later entrant always selects an intermediate level of quality. But the intermediate quality is not endogenously determined and they do not consider the situation where the intermediate quality is offered by an incumbent. Our result on the quality level of the middle quality product is consistent with the prediction of Choi and Shin (1992), who suggest it to be "just over half that of the established firm".

While Constantatos and Perrakis (1997) also find that proliferation may help an established firm to block entry, their focus is on market coverage. In our model, market coverage is endogenously full with or without proliferation. Proliferation intensifies price competition and reallocates demand: in the presence of the middle quality product, price of the high quality product decreases and demand increases, whereas demand for the low quality product drops significantly.

The paper proceeds as follows. Section 2 presents the model for our analysis. Section 2.1 provides a two-quality market environment in which one high quality firm and competitive low quality firms compete in price. We characterize the equilibrium as a benchmark for assessing proliferation incentives later. Section 2.2 evaluates the profitability of proliferation in the existing two-quality market without threat of entry. We determine endogenously the quality choice of proliferation. Section 2.3 examines proliferation as a deterrent when there is threat of entry. Section 3 presents welfare analysis. Section 4 concludes.

2 Model

2.1 Two-quality environment

In a market of quality-differentiated products of the same generic type. Product i is of quality s_i and is sold at price p_i, $i \in \{h, l\}$, where $s_h > s_l$ and $p_h > p_l$. Products with high quality s_h are provided by a single firm, H, and products with low quality s_l are provided by at least two firms, L_1 and L_2. H and the low quality segment engage in maximal differentiation: s_h and s_l are fixed at the two ends of the quality spectrum. The difference between s_h and s_l is denoted as d and is therefore the total length of the spectrum. The unit cost of production c is assumed to be the same for both qualities and is normalized to zero. We assume for now that there is no threat of entry.

There is a continuum of consumers who have identical ordinal preferences over product quality but differ in the willingness to pay. They are uniformly distributed over the interval $[0, \ \bar{\theta}]$ where θ represents their willingness to pay. Each consumer consumes at most one unit of the products and maximises the following utility function

$$U = \begin{cases} \theta s_i - p_i, & if\ consumes, \\ 0, & otherwise. \end{cases} \tag{1}$$

The last consumer with the willingness to pay for the high quality product and the low quality product, is represented by $\theta_h = (p_h - p_l)/d$ and $\theta_l = p_l/s_l$, respectively. That is, consumers with $\theta \in [\theta_h, \bar{\theta}]$ buy the high quality products, consumers with $\theta \in [\theta_l, \theta_h)$ buy the low quality products and consumers with $\theta \in [0, \theta_l)$ buy neither of them.

As L_1 and L_2 offer products with the identical quality s_l, the Bertrand (1883) paradox leads to *Lemma 1*.

Lemma 1 *Price competition drives p_l down to zero. All consumers purchase at least one of the products and the market is covered.*

For H, we can write the optimisation problem as

$$\max_{p_h} \pi_H = \max_{p_h} p_h(\overline{\theta} - \theta_h),$$

where $\overline{\theta} - \theta_h$ is the demand for high quality product. To solve this optimisation problem, we derive the first-order condition with respect to p_h and obtain

$$p_h = \overline{\theta}d/2 + p_l/2,$$

$$D_h = \overline{\theta}d + p_l/2d.$$

Together with *Lemma 1*, we have the following

Lemma 2 *In equilibrium, high and low quality products equally share the market;* $D_h = D_l = \overline{\theta}/2$. *H sets* $p_h = \overline{\theta}d/2$ *and obtains* $\pi_H = \bar{\theta}^2 d/4$, *whereas* L_1 *and* L_2 *obtain zero profit.*

Lemma 2 explains firms' price decisions in the vertically differentiated market. Given that d is constant, the price of the high quality product is higher when the distribution of willingness to pay θ is more dispersed. The market is equally and fully covered by the two existing qualities. Unlike the standard non-cooperative duopoly vertical differentiation model where both high and low quality firms enjoy positive surplus, price competition among products of quality s_l in our model leaves no surplus for the low quality segment.

2.2 Proliferation environment

This section examines H's incentive to proliferate in the existing two-quality market. Since moderate proliferation is more likely to be justified on profitability ground, we allow H to introduce exactly one additional quality, s_m, and assume that

$s_m = \beta s_l + (1 - \beta)s_h, \beta \in (0,1)$.

The above assumption ensures that s_m is of an intermediate level, where β is the location parameter. Given that s_h and s_l are exogenous, β decides s_m. When $\beta > 0.5$, s_m is closer to s_l than to s_h. We exclude the possibility of equality among qualities which is not profitable for H.

Figure 1 illustrates how proliferation may change the market. We know from *Lemmata 1* and *2* that in the market with two qualities s_l and s_h, all consumers will purchase. Consumers with $\theta \in [\theta_l, \theta_h)$ buy products of s_l and consumers with $\theta \in [\theta_h, \bar{\theta}]$ buy products of s_h. When products of s_m are introduced to the market, as shown by the dashed line in Figure 1, consumers face three choices instead of two. *Ceteris paribus*, consumers with $\theta \in [\theta_l, \theta_m)$ buy products of s_l, consumers with $\theta \in [\theta_m, {\theta_h}')$ buy products of s_m, and consumers with $\theta \in [{\theta_h}', \bar{\theta}]$ buy products of s_h.

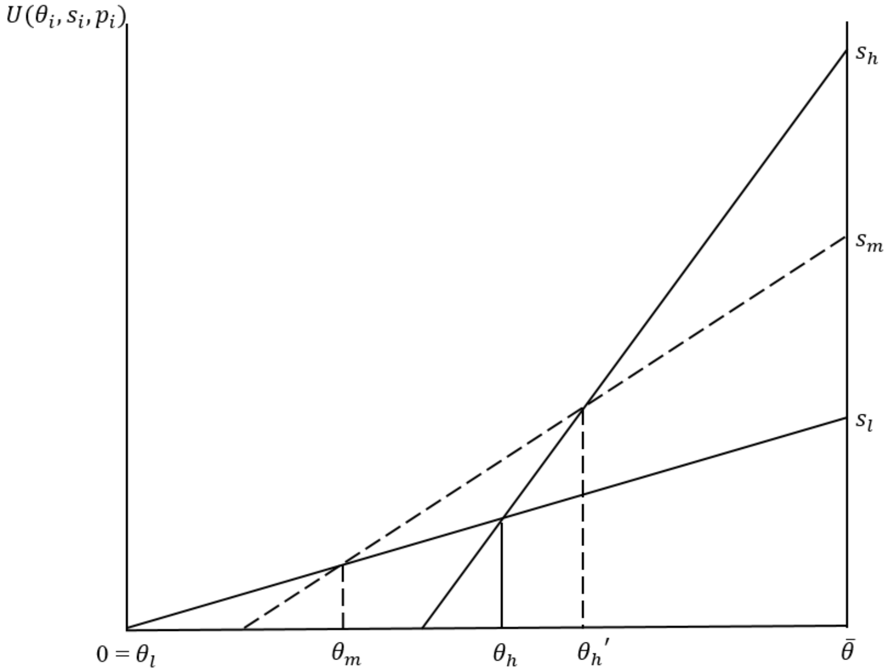

Figure 1 Proliferation in the two-quality market

Johnson and Myatt (2003) suggest that when established firms strategically expand on the quality spectrum, they usually do so by introducing a lower quality product.[6] As $s_h > s_m > s_l$, we have H, rather than one of the low quality firms, to offer s_m. Linking it to the real world, reputation of the designer brands seems to be the key factor contributing to the success of H&M's collaborated ranges. Since in the scope of this paper, proliferation is to be conducted by a single firm, not jointly, it can only be credibly conducted by H. H will proliferate by introducing a middle quality to avoid fierce local competition with the existing qualities, as also suggested by Donnenfeld and Weber (1992).

Upon proliferation, H has to incur some fixed cost f, which may be seen as a product-specific-capital, but the unit cost of production is again normalized to zero. Proliferation works like this: first β is chosen and then all three quality products engage in price competition. As there are two decisions to be made regarding the middle quality product, quality location and price, H faces options of how much control it wishes to take upon proliferation.

	a. Full Control	**b. Semi-full Control**	**c. Independent Control**
$\boldsymbol{\beta}$ **is decided by**	H	H	Middle quality as an independent unit
$\boldsymbol{p_m}$ **is decided by**	H	Middle quality as an independent unit	Middle quality as an independent unit

Table 1 Control options of proliferation

As shown in Table 1, H can choose from three control options. We specify below the optimisation problem of H with each option respectively.

a. Full Control

$$\max_{\beta, p_h, p_m} {\pi_H}^{pro} = \max_{\beta, p_h, p_m} [\pi_h + \pi_m - f]. \tag{2}$$

[6] They provide an example from the Indian watch market in which a launch of a high-end brand by a low-end firm was unsuccessful and eventually exited the market.

With this option, H takes full control over proliferation. It sets the quality location and price of the middle quality product, as well as the price of the original high quality product, so as to jointly maximise its total proliferation profits, π_H^{pro}.

b. Semi-full Control

$$\max_{\beta} \pi_H{}^{pro} = \max_{\beta} \left[\left(\max_{p_h} \pi_h + \max_{p_m} \pi_m\right) - f\right], \tag{3}$$

where

$$\left.\begin{aligned} \max_{p_h} \pi_h &= \max_{p_h} \ p_h D_h \\ \max_{p_m} \pi_m &= \max_{p_m} \ p_m D_m \end{aligned}\right\}. \tag{4}$$

With this option, H sets the location of the middle quality product to maximise total proliferation profits, but does not interfere into price competition. p_h and p_m are chosen to maximise respective unit profits.

c. Independent Control

$$\pi_H{}^{pro} = \max_{p_h} \pi_h + \max_{\beta, p_m} \pi_m - f. \tag{5}$$

With this option, the middle quality product acts purely as an independent unit and has freedom to decide its quality location as well as price, whereas H acts as the parent-firm of the middle quality product and collects the profit that it generates.

To determine whether H has an incentive to proliferate and which control option it will choose, we need to compare $\pi_H{}^{pro}$ under options **a**, **b** and **c** to its profit without proliferation, π_H, as solved in Section 2.1. Specifically, H will proliferate if $\pi_H{}^{pro} > \pi_H$.

Lemma 3 *Under Full Control,* $D_m = 0$ *and* ${\pi_H}^{pro} < \pi_H$ *. H has no incentive to proliferate. Any positive* f*, however small, prevents proliferation from being profitable.*

Proof. Appendix 1.

Lemma 3 says that the market is unaffected by proliferation under Full Control: demand and profit allocations remain the same as in the two-quality environment. When making quality-price decision jointly, H has no incentive to introduce a quality level that is different from the existing qualities.

Lemma 4 *Under Semi-full Control,* $p_m \to 0$ *and* ${\pi_H}^{pro} < \pi_H$ *. H has no incentive to proliferate.*

Proof. Appendix 2.

Under Semi-full Control, the profit maximizing level of β is approaching 1. This means that the difference between s_m and s_l is infinitesimally small and the middle quality product in fact joins the Bertrand competition with the low quality segment. As a result, despite positive D_m, π_m is zero. Again, any positive f would lead to a decrease in π_H if proliferation happens.

Lemma 5 *Under Independent Control,* $\beta = 3/7$ *and* $s_m = 3s_l/7 + 4s_h/7$*. The middle quality product makes a positive profit.*

Proof. Appendix 2.

The middle quality product is closer to but different from the high quality product, which would never be the case under the first two controls. It is intuitive that given $p_l = 0$, s_m needs to be sufficiently differentiated from s_l in order for the middle quality product to be attractive. Among the three control options, Independent Control is the only option with which proliferation actually expands the quality configuration and the proliferated product generates a positive profit. However, it remains to be checked whether proliferation can increase total profits.

Lemma 6 *Under Independent Control,* ${\pi_H}^{pro} = \bar{\theta}^2 d/6 - f < \pi_H$. *H has no incentive to proliferate.*

Proof. Appendix 2.

While ${\pi_H}^{pro} < \pi_H$ for all three control options, Independent Control is the least profitable for H, despite that it is the only option with which $\pi_m > 0$. Table 2 puts together equilibrium outcomes from the two-quality environment and the Independent Control in the proliferation environment. Total demand in the proliferation environment is $\bar{\theta}$, so the market is fully covered with and without proliferation. Through the introduction of the middle quality product, H has successfully broadened its market segment by taking away $3\bar{\theta}/8$ out of the total $4\bar{\theta}/8$ demand that the low quality product would have otherwise captured. Proliferation does not reduce the demand for the high quality product, but p_h is halved as a result, which leads to the fall in H's total profits. Specifically, proliferation creates $\pi_m = \bar{\theta}^2 d/48$ but decreases π_h from $12\bar{\theta}^2 d/48$ to $7\bar{\theta}^2 d/48$, therefore is not desirable for H.

Environment	**Price**			**Demand**			**Profit**		
	p_l	p_m	p_h	D_l	D_m	D_h	π_l	π_m	π_h
Two-quality	0	-	$\bar{\theta} d/2$	$\bar{\theta}/2$	-	$\bar{\theta}/2$	0	-	$\bar{\theta}^2 d/4$
Proliferation (IC)	0	$\bar{\theta} d/14$	$\bar{\theta} d/4$	$\bar{\theta}/8$	$7\bar{\theta}/24$	$7\bar{\theta}/12$	0	$\bar{\theta}^2 d/48$	$7\bar{\theta}^2 d/48$

Table 2 Two-quality environment *vs.* proliferation environment with independent control

Lemmata 3, *4* and *6* lead to *Proposition 1*.

Proposition 1 *In the absence of threat of entry, proliferation is not profitable for H under any level of control. Ceteris paribus, H will not initiate proliferation.*

Given that the most moderate level of proliferation of offering only one additional quality cannot be justified on profitability ground, *Proposition 1* also implies the undesirability of excessive proliferation. The result suggests the stability of the principle of maximal differentiation in the market configuration featured in this paper. This is however, conditional on no entry threats. Since proliferation is also a candidate for deterrence, it may be optimal for H to do so when the condition is relaxed.

2.3 Entry environment

This section examines H's incentive to proliferate in the existing two-quality market, so as to deter a potential entrant E. Upon entry, E makes quality-price decision to ensure positive post-entry surplus.

Lemma 7 *E will enter the market with products of quality $s_E = 3s_l/7 + 4s_h/7$. E sets $p_E = \overline{\theta}d/14$ and obtains $\pi_E = \bar{\theta}^2 d/48 - k$, where $k \in [0, \bar{\theta}^2 d/48]$ is the fixed entry cost.*

Since $\pi_E > 0$, E always has an incentive to enter. The quality-price decision made by E is the same as that under Independent Control in the proliferation environment. This is because when making decisions, the independent unit maximises its own profit (and not the total profits of its parent-firm) just as what an entrant would do. The only difference is that π_m generated under Independent Control contributes to the total profits of the parent-firm, ${\pi_H}^{pro}$, whereas E gets to keep π_E for itself.

Suppose that given the distribution of θ, it is never profitable for E to enter with a fourth quality, then anticipating the quality-price decision of E, H has an incentive to proliferate to deter entry if doing so is more profitable than accommodating entry, i.e., if ${\pi_H}^{pro} > {\pi_H}^{E}$.

Proposition 2 *In the presence of threat of entry, H's incentive towards proliferation with Independent Control depends on the value of proliferation cost, f, such that*

- ${\pi_H}^{pro} < {\pi_H}^{E}$ for $f \in (\bar{\theta}^2 d/48, \infty)$, *proliferation is not profitable for* H*;*
- ${\pi_H}^{pro} > {\pi_H}^{E}$ for $f \in [0, \bar{\theta}^2 d/48)$, H *has an incentive to proliferate and achieve a second best profit.*

Proof: When considering proliferation to deter entry, H will choose Independent Control because with this control option, the proliferated product is of quality $s_m = s_E$, thus can restrict the market space available and make it impossible for E to enter,[7] whereas with the other two control options, proliferation does not change the quality configuration in the market. Table 3 represents firms' profits in different environments.

Environment	Profit of firms		
	$\boldsymbol{H}$	$\boldsymbol{E}$	$\boldsymbol{L_1, L_2}$
Two-Quality	$12\bar{\theta}^2 d/48$	-	0
Proliferation (IC)	$8\bar{\theta}^2 d/48 - f$	0	0
Entry	$7\bar{\theta}^2 d/48$	$\bar{\theta}^2 d/48 - k$	0

Table 3 Firms' profits in two-quality, proliferation (IC) and entry environments

Proposition 2 is obtained by comparing H's profit in proliferation environment and entry environment. As shown in Table 3, H's incentive to proliferate anticipating the decision of E depends on the magnitude of proliferation cost.

Propositions 1 and *2* together suggest that, when $f > \bar{\theta}^2 d/48$, proliferation is not profitable for H, regardless of whether there is threat of entry. However, when $f \leq \bar{\theta}^2 d/48$, proliferation becomes optimal only if it is used as an abusive exclusionary conduct. That is, even the most moderate level of proliferation of introducing one additional quality may be anticompetitive.

[7] In this paper, we focus on whether the high quality incumbent has the incentive to proliferate to deter entry. See, e.g., Dixit (1980) and Judd (1985) for the issue of credibility in entry deterrence.

3 Welfare analysis

This section examines how consumer surplus and total welfare change when the market has three instead of two qualities. Consumer surplus is generated from the consumption of each quality. In the two-quality market, consumer surplus consists of CS_h from consumers with $\theta \in [\theta_h, \bar{\theta}]$ and CS_l from consumers with $\theta \in [\theta_l, \theta_h)$, where

$$\left.\begin{aligned} CS_h &= \int_{\theta_h}^{\bar{\theta}} (\theta s_h - p_h)\, d\theta \\ CS_l &= \int_{\theta_l}^{\theta_h} (\theta s_l - p_l) d\theta \end{aligned}\right\}. \tag{6}$$

In the three-quality market, consumer surplus consists of CS_h from consumers with $\theta \in [\theta_h, \bar{\theta}]$, CS_m from consumers with $\theta \in [\theta_m, \theta_h)$ and CS_l from consumers with $\theta \in [\theta_l, \theta_m)$, where

$$\left.\begin{aligned} CS_h &= \int_{\theta_h}^{\bar{\theta}} (\theta s_h - p_h)\, d\theta \\ CS_m &= \int_{\theta_m}^{\theta_h} (\theta s_m - p_m)\, d\theta \\ CS_l &= \int_{\theta_l}^{\theta_m} (\theta s_l - p_l) d\theta \end{aligned}\right\}. \tag{7}$$

Calculation. See Appendix 3.

Summing up consumer surplus and firms' profits, we are able to compare total welfare in different environments, as shown in Table 4. Since the third quality could be offered by proliferation or by the entrant, we include both of them for comparison. Proliferation and entry achieve the same consumer surplus, which is greater than the consumer surplus in the two-quality market, whereas they both decrease industry profits.

Environment	Welfare Comparison		
	Consumer surplus	**Industry profits**	**Total welfare**
Two-quality	$\overline{\theta}^2 s_h/8 + 3\overline{\theta}^2 s_l/8$	$\overline{\theta}^2 d/4$	$3\overline{\theta}^2 s_h/8 + \overline{\theta}^2 s_l/8$
Three-quality (Proliferation)	$119\overline{\theta}^2 s_h/288 + 91\overline{\theta}^2 s_m/1152 + \overline{\theta}^2 s_l/128 - \overline{\theta}^2 d/6$	$\overline{\theta}^2 d/6 - f$	$119\overline{\theta}^2 s_h/288 + 91\overline{\theta}^2 s_m/1152 + \overline{\theta}^2 s_l/128 - f$
Three-quality (Entry)	$119\overline{\theta}^2 s_h/288 + 91\overline{\theta}^2 s_m/1152 + \overline{\theta}^2 s_l/128 - \overline{\theta}^2 d/6$	$\overline{\theta}^2 d/6 - k$	$119\overline{\theta}^2 s_h/288 + 91\overline{\theta}^2 s_m/1152 + \overline{\theta}^2 s_l/128 - k$

Table 4 Welfare comparison

Lemma 2.7 *Moving from two-quality to three-quality market, consumer surplus strictly increases and industry profits strictly decrease, regardless of values of f and k.*

Proof. Appendix 4.

Following *Lemma 7*, whether the rise in consumer surplus outweighs the fall in producer surplus depends on the fixed cost upon having the middle quality product.

Proposition 3 *Moving from two-quality to three-quality market, the change in total welfare is such that*

- *When the third quality is introduced through H's proliferation, total welfare increases if and only if* $f \in [0, (44s_h + 91s_m - 135s_l)\overline{\theta}^2/1152)$*;*
- *When the third quality is introduced by* E*, total welfare increases if and only if* $k \in [0, (44s_h + 91s_m - 135s_l)\overline{\theta}^2/1152)$.

Proof. Appendix 4.

When the fixed cost is smaller than the critical threshold, having the middle quality product is welfare-enhancing; otherwise total welfare decreases as a result of a large fall in industry profits. Proliferation and entry affect total welfare in the identical way if $f = k$. From a welfare point of view, whether the third quality should be offered by H or E depends on their relative cost of introducing it. If H benefits from the fact that it is already established therefore incurs lower cost, then not only proliferation can increase welfare, it also does in a way that is more efficient than entry. This leads to an interesting result that, despite being an exclusionary conduct carried out deliberately by a dominant firm facing threat of entry, proliferation always benefits consumers and can even increase total welfare.

4 Conclusion

Proliferation in horizontal differentiation markets is well understood. As proliferation is becoming increasingly popular in vertical differentiation markets, and many vertically differentiated brands, especially those in the sportswear and fashion apparel industry, have been repeatedly engaging in joint-proliferations, e.g., H&M collaborations, it seems important to investigate proliferation in vertical differentiation markets. This paper provides some initial steps towards this goal.

We offer an evaluation of proliferation by assessing its effects on profitability, entry deterrence and welfare in a simple yet intuitive framework. We highlight the trade-off between softening competition and broadening market segments brought about by proliferation and show that even the most moderate level of proliferation may be anticompetitive. Nonetheless when proliferation is carried out as a division that competes independently with the parent-firm, it always benefits consumers and could even increase total welfare in a way that is more efficient than entry. In addition, we have extended the literature on entrant choice in duopoly markets following the principle of maximal differentiation by endogenously determining the quality level of the additional middle quality product.

In this paper we focus on proliferation incentives of the high quality firm. The current framework, while useful for addressing our research interests, does not allow the low quality segment to play a positive role. To move closer to the real world and replicate joint-proliferation, future research should introduce competition to the high quality segment and horizontal differentiation among high quality firms and low quality firms, and assess the respective incentives of vertically related firms to collaborate. For example, a low quality firm may find it desirable to collaborate with a high quality firm so as to differentiate itself from other low quality firms.

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Appendices

1. Proof of *Lemma 3*

Expand H's optimisation problem (2),

$$\pi_H^{pro} = p_h\left(\overline{\theta} - \frac{p_h - p_m}{\beta d}\right) + p_m\left[\frac{p_h - p_m}{\beta d} - \frac{p_m}{(1-\beta)d}\right] - f.$$

Derive the first order conditions with respect to p_h and p_m

$$\frac{\partial \pi_H^{pro}}{\partial p_h} = \overline{\theta} - \frac{2p_h}{\beta d} + \frac{2p_m}{\beta d} = 0,$$

$$\frac{\partial \pi_H^{pro}}{\partial p_m} = \frac{2p_h}{\beta d} - \frac{2p_m}{\beta d} - \frac{2p_m}{(1-\beta)d} = 0.$$

The second-order conditions are fulfilled. Prices and demands are $p_h = \bar{\theta}d/2$ and $p_m = \bar{\theta}d(1-\beta)/2$; $D_h = \bar{\theta}/2$, $D_m = 0$ and $D_l = \bar{\theta}/2$. Under Full Control, demand allocation stays the same as in the two-quality environment. □

2. Proofs of *Lemmata 4, 5* and *6*

We first look at independent price competition as in (4). Derive the first-order conditions respectively and solve for prices, demands and profits as functions of β and $\bar{\theta}$

$$p_h = \frac{2\overline{\theta}\beta d}{3+\beta},$$

$$p_m = \frac{\overline{\theta}\beta d(1-\beta)}{3+\beta},$$

$$D_h = \frac{2\overline{\theta}}{3+\beta},$$

$$D_m = \frac{\overline{\theta}}{3+\beta},$$

$$D_l = \frac{\overline{\theta}\beta}{3+\beta},$$

$\pi_h = \frac{4\bar{\theta}^2 \beta d}{(3+\beta)^2}$,

$\pi_m = \frac{\bar{\theta}^2 \beta d(1-\beta)}{(3+\beta)^2}$,

$\pi_l = 0$.

Under Semi-full Control, β is set by H to maximise $\pi_h + \pi_m$. We can find that, for $\beta \in (0, 1)$, $\pi_h + \pi_m$ is increasing in β; $\partial(\pi_h + \pi_m)/\partial\beta = \bar{\theta}^2 d(15 - 11\beta)/(3 + \beta)^2 > 0$. Hence $\pi_h + \pi_m$ is maximised at $\beta \to 1$. When $\beta \to 1$, $s_m \to s_l$, $p_h \to \bar{\theta} d/2$, which is the same in the two-quality environment, and $p_m \to 0$.

Under Independent Control, β is set to maximise the unit profit of the middle quality product, not total profits

$$\frac{\partial \pi_m}{\partial \beta} = \frac{(3+\beta)^2(\bar{\theta}^2 d - 2\bar{\theta}^2 \beta d) - 2\bar{\theta}^2 \beta d(1-\beta)(3+\beta)}{(3+\beta)^4} = 0.$$

Solve for the above equation and $\beta = 3/7$. The second-order conditions are fulfilled. The equilibrium outcomes are presented in Table 2 in Section 2.2. □

3. Calculation of equations (6) and (7)

Expand and solve for (6), in the two-quality market

$CS_h = \frac{1}{2}\overline{\theta}^2 s_h - \overline{\theta} p_h - \frac{1}{2}{\theta_h}^2 s_h + \theta_h s_h = \frac{3}{8}\overline{\theta}^2 s_h - \frac{1}{4}\overline{\theta}^2 d$,

$CS_l = \frac{1}{2}{\theta_h}^2 s_l = \frac{1}{8}\overline{\theta}^2 s_l$,

$CS = CS_l + CS_h = \frac{1}{8}\overline{\theta}^2 s_h + \frac{3}{8}\overline{\theta}^2 s_l$.

Expand and solve for (7), in the three-quality market

$CS_h = \frac{1}{2}\overline{\theta}^2 s_h - \overline{\theta} p_h - \frac{1}{2}{\theta_h}^2 s_h + \theta_h s_h = \frac{119}{288}\overline{\theta}^2 s_h - \frac{7}{48}\overline{\theta}^2 d$,

$$CS_m = \frac{1}{2}{\theta_h}^2 s_m - \theta_h p_m - \frac{1}{2}{\theta_m}^2 s_m + \theta_m s_m = \frac{91}{1152}\overline{\theta}^2 s_m - \frac{1}{48}\overline{\theta}^2 d,$$

$$CS_l = \frac{1}{2}{\theta_m}^2 s_l = \frac{1}{128}\overline{\theta}^2 s_l,$$

$$CS = CS_l + CS_m + CS_h = \frac{119}{288}\overline{\theta}^2 s_h + \frac{91}{1152}\overline{\theta}^2 s_m + \frac{1}{128}\overline{\theta}^2 s_l - \frac{1}{6}\overline{\theta}^2 d.$$

□

4. Proofs of *Lemma 7* and *Proposition 3*

Moving from two-quality to three-quality market, consumer surplus strictly increases if

$$\frac{119}{288}\overline{\theta}^2 s_h + \frac{91}{1152}\overline{\theta}^2 s_m + \frac{1}{128}\overline{\theta}^2 s_l - \frac{1}{6}\overline{\theta}^2 d > \frac{1}{8}\overline{\theta}^2 s_h + \frac{3}{8}\overline{\theta}^2 s_l.$$

That is, if

$$\frac{140}{1152}s_h + \frac{91}{1152}s_m > \frac{231}{1152}s_l,$$

which always holds since

$$\frac{140}{1152}s_h + \frac{91}{1152}s_m > \frac{140}{1152}s_m + \frac{91}{1152}s_m,$$

and

$$\frac{231}{1152}s_m > \frac{231}{1152}s_l.$$

It is straightforward to verify that, moving from two-quality to three-quality market, industry profits decrease. Because $1/4 > 1/6$,

$$\frac{1}{4}\overline{\theta}^2 d > \frac{1}{6}\overline{\theta}^2 d - f,$$

and

$$\frac{1}{4}\overline{\theta}^2 d > \frac{1}{6}\overline{\theta}^2 d - k.$$

Proliferation increases total welfare if

$$\frac{119}{288}\bar{\theta}^2 s_h + \frac{91}{1152}\bar{\theta}^2 s_m + \frac{1}{128}\bar{\theta}^2 s_l - f > \frac{3}{8}\bar{\theta}^2 s_h + \frac{1}{8}\bar{\theta}^2 s_l.$$

That is, if

$$f < \frac{44}{1152}\bar{\theta}^2 s_h + \frac{91}{1152}\bar{\theta}^2 s_m - \frac{135}{1152}\bar{\theta}^2 s_l.$$

It is straightforward to verify that the right hand side of the above inequation is strictly positive.

Likewise, entry increases total welfare if

$$k < \frac{44}{1152}\bar{\theta}^2 s_h + \frac{91}{1152}\bar{\theta}^2 s_m - \frac{135}{1152}\bar{\theta}^2 s_l.$$

It follows that, total welfare is higher with proliferation than with entry if and only if $f < k$. □