On the use of Singular Spectrum Analysis for Forecasting U.S. Trade before, during and after the 2008 Recession*

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Abstract

This paper is aimed at introducing the powerful, nonparametric time series analysis and forecasting technique of Singular Spectrum Analysis (SSA) for trade forecasting via an application which evaluates the impact of the 2008 recession on U.S. trade forecasting models. This research is felicitous given the magnitude of the structural break visible in the U.S. trade series following the 2008 economic crisis. Structural breaks resulting from such recessions might affect conclusions from traditional unit root tests and forecasting models which makes use of these tests. As such, it is prudent to evaluate the sensitivity and reliability of parametric, historical trade forecasting models in comparison to the relatively modern, nonparametric models. In doing so, we introduce the SSA technique for trade forecasting and perform exhaustive statistical tests on the data for normality, stationarity and change points, and the forecasting results for statistical significance prior to reaching the well-founded conclusion that SSA is less sensitive to the impact of recessions on U.S. Trade, in comparison to an optimized ARIMA model, Exponential Smoothing and Neural Network models. Ergo, we conclude that SSA is able to provide more accurate forecasts for U.S. Trade in the face of recessions, and is therefore presented as an apt alternative for U.S. Trade forecasting before, during and after a future recession.

Keywords: United States; international trade; recession; forecasting; Singular spectrum analysis.
JEL: F1, F10, F17.

1 Introduction

Globalization in the modern age continues to augment the importance of international trade towards the smooth functioning of any given economy. In a highly interconnected world, international trade has a direct impact not only on the economic growth of trading partners, but also on the determination of exchange rates. For these reasons, and given the prevailing economic climate around the globe, trade analysis remains a hot topic and we believe it is opportune to analyse the effectiveness of trade forecasting models amidst the impact of recessions such as the 2008 financial crisis which appears to have left a major break in the U.S. Trade series.

This study is motivated by the challenge of introducing and ascertaining the effectiveness of Singular Spectrum Analysis (SSA) for trade forecasting before, during and after recessions

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in comparison to other widely used trade forecasting models. To achieve this objective, we consider the United States (U.S.) imports and exports series. The choice of the U.S. economy was influenced to a lesser extent by the U.S.' continuing stance as a global economic superpower, and to a greater extent by the history of trading patterns in the U.S., and most importantly the impact of the 2008 recession which created a major structural break in the U.S. Trade series (see, Figure 1). Structural breaks of such major magnitude are likely to have adverse impacts on forecasting models. In fact, a closer look at Figure 1 enables us to identify a second structural break of a comparatively smaller magnitude caused by the 2001 recession, further justifying the selection of U.S. trade as the data set for this research.

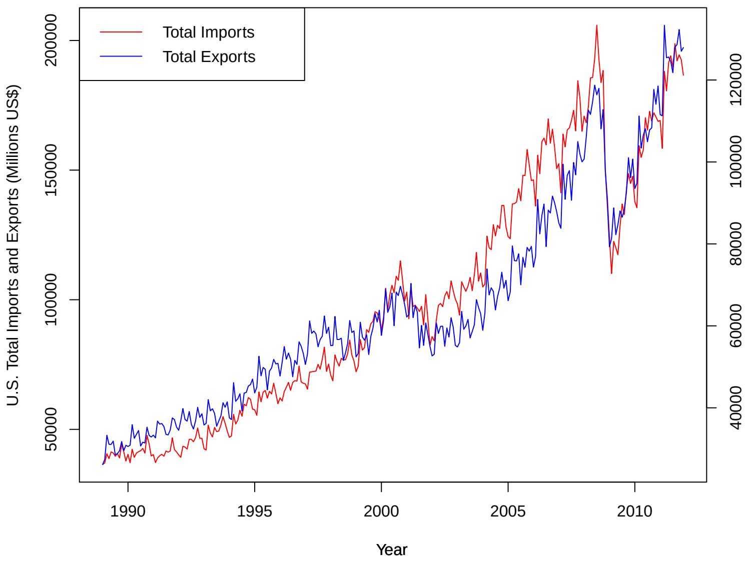

Figure 1: U.S. Imports and Exports series (1989-2011)[1].

Krugman and Baldwin (1987) asserts the U.S. was a net exporter in the 1950's and the 1960's. Since then, the U.S. trade position has deteriorated continuously. The United States Census Bureau indicates that the most recent U.S. trade surplus was recorded in the year 1970. Krugman and Baldwin (1987), through their exemplary work, identified the major strain imposed by persistent trade deficits on U.S. economic policy along with an in-depth analysis of the continuous trade deficits.Interestingly, forty decades following the 1970 trade surplus the U.S. continues to experience prolonged and persistent trade deficits (as verified by the Trade data published online at the U.S. Department of Commerce and the U.S. Census Bureau). Accordingly, it is not surprising that trade analysis and forecasting continues to remain a core concern for policy makers.

Over the years a variety of forecasting models have been evaluated for trade forecasting. A concise report on trade forecasting models which have been utilised up until the late 1970's and their limitations can be found in Coccari (1978). In the past, as it is today, statisticians, academics, analysts and policy makers have constantly endeavoured to obtain the most accurate trade forecasts possible, so as to enable finalization of crucial policy decisions which can aid in lucrative resource allocations. This ongoing effort to improve the accuracy of trade forecasts

[1]Data source: http://www.bea.gov/international/index.htm.

have resulted in the rapid development of trade forecasting models over time. Initially, such models were restricted to basic, linear, parametric models which have now been extended to more complex, nonparametric models. Whilst it is not the objective of this paper to review all trade forecasting models, we provide a brief review into the applications of ARIMA, Exponential Smoothing and Neural Networks for forecasting trade.

Autoregressive Integrated Moving Averages (ARIMA) have been adopted for trade forecasting both historically and in the recent past (see for example, Dale and Bailey, (1982); Espasa and Pena, (1995); Akhtar, (2003); Keck and Raubold, (2006); Kargbo, (2007); Narayan, Narayan and Prasad, (2008); Keck, Raubold, and Truppia, (2009); Emang et al., (2010); Khan, (2011); Sahu and Mishra, (2013)). ARIMA is one of the many parametric time series analysis and forecasting techniques which are used for trade forecasting. In this paper we use an optimized ARIMA model as a benchmark. Parametric models are restricted by the assumptions of stationarity and normality which are unlikely to hold in a real world scenario, especially following recessions which makes a time series non-stationary. The Exponential Smoothing (ETS) technique was developed through the work of Holt (1957), Brown (1959) and Winters (1960). ETS is also a parametric forecasting technique which has been considered for trade forecasting in the recent past (see, for example, Kargbo, (2007); Co and Boosarawongse, (2007); Ostertagova and Ostertag, (2012)). Hooper (1976) pointed out the disadvantages of relying on parametric models as these resulted in a significant deterioration of the forecast accuracy associated with U.S. trade models resulting from their inability of capturing the rapid changes occurring in the economic environment. However, given the very recent applications of parametric models for forecasting trade, the consideration of such models in this paper is justified.

Recently there has been an increase in the use of nonparametric models for trade forecasting. Nonparametric models have the advantage of not being restricted by any of the parametric assumptions which in turn enables a much closer representation of the real world scenario. Neural Networks have been a popular choice of nonparametric models for trade forecasting. For example, Co and Boosarawongse (2007) finds Neural Networks (NN) outperforming ARIMA and ETS at forecasting rice exports in Thailand whilst Pakravan et al. (2011) finds a feed-forward NN model outperforming recurrent networks and multilayer perceptron networks at forecasting Iranian rice imports. In addition to the articles which have been cited above, there has been a surge in trade forecasting literature following the 2008 recession suggesting a renowned interest in this area of research. Chou et al. (2008), Burgert and Des (2008), Xie and Xie (2009), Stoevsky (2009) and, Lutero and Marini (2010) are few such examples.

The literature also highlights issues beyond problems associated with trade forecasting models and difficulties relating to data collection. Historically, one of the major predicaments with trade forecasting was the calculation of the forecast error (Major, 1967). However, through statistical maturity issues pertaining to the calculation of forecast errors have been overcome via metrics such as the Mean Absolute Percentage Error (MAPE), Mean Squared Error (MSE) and the Root Mean Squared Error (RMSE) which are now utilized as standard measures for calculating forecast accuracy (see for example, Keck and Raubold (2006); Hassani et al. (2014)).

In this paper we introduce a nonparametric forecasting technique known as SSA for trade forecasting. As the introduction of SSA for trade forecasting is the primary aim of this paper, we believe it is pertinent to comment on the advantages of using such a method. We begin by considering the power and performance of SSA. According to the literature, SSA has recently been applied successfully to solve many practical problems (see for example, Ghil et al., (2002); Hassani et al. (2009,2012;2013a,b;2015); Hassani and Thomakos (2010); Beneki and Silva, (2013); Silva (2013)). These applications are diverse and cover not only the modelling and forecasting of economic and financial time series but also energy, medical and environmental problems. As such, it is evident that the SSA technique is growing into a powerful and widely used technique which performs considerably well in a variety of fields. Secondly, the SSA model is nonparametric which means that it is not limited by the restrictive parametric assumptions relating to the nature of the data, i.e. linearity, normality and stationarity. This is highly useful in real world scenarios as it enables users to model without the need for data transformations which according to Hassani et al. (2013a) results in a loss of information. Thirdly, the SSA technique has some similarities with autoregressive modelling and those interested are directed to Hassani and Thomakos (2010) for a detailed explanation on the similarities whilst a brief explanation is provided at the end of Appendix B. Fourthly, the SSA technique also has the option of incorporating change point detection which is not pursued in this paper as here we intend on attaining certain key insights into the best approach for modelling U.S. trade with change point detection in the future. In addition, as this paper marks the introduction of SSA for trade forecasting we wish to consider the most basic version of SSA with the likes of ARIMA, ETS and NN which are popular time series analysis and forecasting methods which do not incorporate change point detection. Finally, it is worth noting the modelling approach of SSA in comparison to the classical time series analysis and forecasting models. In brief, the classical methods forecast both the signal and noise found in a time series whereas SSA seeks to filter the noise and forecast the approximated signal. For these reasons it is evident that international trade modelling and forecasting can benefit through the application of a method such as SSA which is proposed through this paper.

In terms of using SSA for forecasting before, during and after recession, Hassani et al. (2013b) through a recent collaboration with the Office for National Statistics in UK evaluated forecasting eight British economic indexes before, during and after recessions using ARIMA, Holt-Winters and SSA, and found the SSA model outperforming ARIMA and Holt-Winters. Therefore, here we do not provide major emphasis to the popular time series forecasting method of Holt-Winters as it has been proven on earlier occasions that SSA outperforms Holt-Winters in terms forecasting accuracy (see for example, Hassani, (2007); Hassani et al. (2009,2012); Hassani et al. (2013)). However, ARIMA is considered as in this paper we use an optimized version of the ARIMA model. In this study, we also evaluate the applicability of the Hassani et al. (2013) approach of using a small trajectory matrix during the recession by applying the same approach for forecasting U.S. Trade. Accordingly, we use ARIMA, ETS, NN and SSA to forecast U.S. imports and exports and then evaluate the impact of the 2008 recession on the forecasting accuracy of these models which represent both parametric and nonparametric forecasting techniques. In general, structural breaks such as the one caused by the on-set of the 2008 recession results in making the time series non-stationary in mean and variance.

The remainder of this paper is organised as follows. Section 2 describes the forecasting methods of ARIMA, Exponential Smoothing, Neural Networks and SSA. Section 3 presents the data and Section 4 is dedicated towards the empirical results. The paper concludes with a discussion and summary in Section 5.

2 Forecasting Methods

2.1 Autoregressive Integrated Moving Average (ARIMA)

An optimal version of Box and Jenkins (1970) ARIMA model is used as a benchmark in this paper. The variation is referred to as Automatic-ARIMA and is provided through the forecast package for the *R* software. For a detailed description of the algorithm on which Automatic-ARIMA is based, see Hyndman and Khandakar (2008). The number of differences is defined as d, and the first step is the determination of its value using KPSS tests in (Kwiatkowski et al., 1992). Next, the algorithm minimises the Akaike Information Criterion (AIC) to determine the values of p and q. The optimal model is chosen to be the model which represents the smallest AIC from the following options: ARIMA*(2,d,2)*, ARIMA*(0,d,0)*, ARIMA*(1,d,0)* and ARIMA*(0,d,1)*. The decision on the inclusion or exclusion of the constant c is made depending on the value of d. Note that where necessary, Box-Cox transformations have been used on the data by setting the parameter $\lambda = 0$ in order to comply with the parametric assumptions which govern ARIMA.

To expand on the above summary, we provide the following equations based on Hyndman and Athanasopoulos (2013). A non-seasonal ARIMA model may be written as:

$$(1 - \phi_1 B - \dots \phi_p B^p)(1 - B)^d y_t = c + (1 + \phi_1 B + \dots + \phi_q B^q) e_t, \tag{1}$$

or

$$(1 - \phi_1 B - \dots \phi_p B^p)(1 - B)^d (y_t - \mu t^d / d!) = (1 + \phi_1 B + \dots + \phi_q B^q) e_t, \tag{2}$$

where μ is the mean of $(1 - B)^d (y_t$, $c = \mu(1 - \phi_1 - \dots - \phi_p)$ and B is the backshift operator. In the R software, the inclusion of a constant in a non-stationary ARIMA model is equivalent to introducing a polynomial trend of order d in the forecast function. It should be noted that when d=0, μ is the mean of y_t.

According to Hyndman and Khandakar (2008), the seasonal ARIMA model can be expressed as:

$$\Phi(B^m)\phi(B)(1 - B^m)^D (1 - B)^d y_t = c + \Theta(B^m)\theta(B)\epsilon_t, \tag{3}$$

where $\Phi(z)$ and $\Theta(z)$ are the polynomials of orders P and Q, and ϵ_t is white noise. Note that if c $\neq 0$, there is an implied polynomial of order $d + D$ in the forecast function. As mentioned previously, to determine the values of p and q the AIC of the following form is minimised:

$$AIC = -2log(L) + 2(p + q + P + Q + k), \tag{4}$$

where $k = 1$ if c $\neq 0$ and 0 otherwise, and L represents the maximum likelihood of the fitted model.

The process for obtaining point forecasts using the R software is concisely explained in Hyndman and Athanasopoulos (2013) as follows. Firstly, we expand the relevant ARIMA equation (i.e. non-seasonal or seasonal) so that y_t is on the left hand side with all other terms on the right. Thereafter, we rewrite the ARIMA equation by replacing t with $T + h$ and finally, on the right hand side of this equation we simply replace future observations by their forecasts, future errors by zero, and past errors by the corresponding residuals. Then, beginning with the forecasting horizon $h = 1$ month ahead, we repeat these steps for $h = 3$, 6, and 12 months ahead until all forecasts have been calculated.

Note that in terms of determining the number of differences d, required for the ARIMA modelling process, the algorithm allows one to select this value using three different approaches; KPSS unit root tests, Augmented Dickey-Fuller (ADF) test or the Phillips-Perron (PP) unit root tests. KPSS tests led to better forecasts in comparison to the ADF and PP tests when applied to the M3 competition data (Hyndman 2014). However, instead of relying on these results alone, in this paper we consider modelling using all three approaches and report the results based on the KPSS unit root tests from Kwiatkowski et al. (1992) for the number of differences d required as it provided better forecasts in comparison to ARIMA models considering ADF and PP tests. For U.S. trade forecasting, ARIMA uses the following differencing for both imports and exports; $d = 0$ before and during the 2008 recession, and $d = 1$ after the 2008 recession. Furthermore, we have also evaluated both AIC and Bayesian Information Criterion (BIC) for determining the best p and q for each ARIMA model and report the results based on KPSS tests and AIC as it provided the better forecasts.

2.2 Exponential Smoothing (ETS)

The ETS technique incorporates the foundations of exponential smoothing and is made available through the forecast package for the R software. ETS overcomes a limitation found in earlier exponential smoothing models which did not provide a method for easy calculation of prediction intervals (Makridakis, Wheelwright and Hyndman, 1998). The ETS model from the forecast package considers the error, trend and seasonal components along with over 30 possible options for choosing the best exponential smoothing model via optimization of initial values and parameters using the maximum likelihood estimator and selecting the best model based on the AIC. A detailed description of ETS can be found in Hyndman and Athanasopoulos (2013).

Figure 2 in Appendix A summarises in table format all the ETS formula's that are evaluated in the forecast package to select the best model to fit the data. Note that in this figure, ell_t denotes the series level at time t, b_t denotes the slope, s_t denotes the seasonal component of the series, and m denotes the number of seasons in a year; α, β, γ and ϕ are smoothing parameters, $\phi_h = \phi + \phi_2 + \ldots + \phi^h$ and $h_m^+ = [(h-1)modm] + 1$ (Hyndman and Athanasopoulos, 2013).

2.3 Neural Networks (NN)

The NN model used in this research is a system of feed-forward neural networks with lagged inputs and one hidden layer. A detailed description of the model can be found in Hyndman and Athanasopoulos (2013) along with an explanation on the underlying dynamics. In brief, the *nnetar* function trains 25 neural networks by adopting random starting values and then obtains the mean of the resulting predictions to compute the forecasts. The neural network takes the form

$$\hat{y}_t = \hat{\beta}_0 + \sum_{j=1}^{k} \hat{\beta}_j \psi(x_t^{'}.\hat{\gamma}_j), \tag{5}$$

where x_t consist of p lags of y_t and the function ψ has the logistic form

$$\psi(x_t^{'}.\hat{\gamma}_j) = [1 + exp(-\hat{\gamma}_{j0} + \sum_{i=1}^{p} \hat{\gamma}_{ji}.y_{t-1})]^{-1} j = 1, \ldots, k \tag{6}$$

This form of neural networks is referred to as a one hidden layer feed forward neural network model. The nonlinearity arises through the lagged y_t entering in a flexible way through the logistic functions of (6). The number of logistic functions (k) included, is known as the number of hidden nodes.

The neural network models are estimated using an automatic forecasting model known as *nnetar* which is provided through the forecast package in R. For a detailed explanation on how the nnetar model operates, see Hyndman et al. (2013). The parameters in the neural network model are selected based on a loss function embedded into the learning algorithm. This loss function could be for example the Root Mean Square Error (RMSE) which is also adopted in the SSA algorithm explained below. The *nnetar* function trains 25 networks by using random starting values and then obtains the average of the resulting predictions to compute the forecast. It may be noted that in all cases the selected neural network model has only k=1 hidden node, p=2 lags and we adopt annual difference specifications.

2.4 Singular Spectrum Analysis (SSA)

The roots of SSA are closely associated with Broomhead and King (1986). A detailed description of the methodology of SSA can be found in Golyandina et al. (2001), and Hassani (2007). As mentioned in Hassani (2007), the SSA technique is made up of two stages, each with two steps. Stage 1 is referred to as decomposition and consists of the two steps known as embedding and singular value decomposition (SVD). Stage 2 is referred to as reconstruction, and consists of the grouping and diagonal averaging steps. The SSA technique has the advantage of being nonparametric. As such it is not restricted by the assumptions of stationarity which governs the parametric models. This is important as structural breaks are infamous for making a time series non-stationary. With SSA there is no requirement to test the data for stationarity and it can handle both stationary and non-stationary time series (Hassani et al. 2013). SSA's capabilities at handling structural breaks are well documented in Golyandina et al. (2001, Ch. 3) and is therefore not reproduced in detail here. In brief, an advanced version of SSA enables the incorporation of change point detection (for early detection of structural breaks) which is not pursued in this paper so as to ensure that all models compared are on an equal platform. SSA change point detection should not be mistaken for a structural break test as it does not give a significance value, however it relies on SVD and the trajectory matrix for early detection of structural breaks in a time series. It should be noted that SSA has two variations known as Vector SSA (VSSA) and Recurrent SSA. In this paper we adopt the basic Vector SSA method with optimal SSA choices (Hassani and Mahmoudvand, 2013) to model and forecast U.S. Trade. This is because in the presence of structural breaks, Vector SSA is known to be more robust than Recurrent SSA (see for example, Golyandina et al. (2001)). In Appendix B we present the VSSA forecasting algorithm used in this paper[2] and in doing so we mainly follow Hassani et al. (2015).

3 U.S. Imports and Exports Data

The data used in this study were obtained via the U.S. Department of Commerce: Bureau of Economic Analysis and includes monthly imports and exports of goods from January 1989 to December 2011. The Balance of Payments based, seasonally unadjusted data has been selected for this study. The main reason for selecting seasonally unadjusted data was to ensure the SSA technique does not have any undue advantage over the other parametric models such as ARIMA. This is important as in previous research, Abeysinghe (1994) found that seasonal dummies result in poor forecasts while Hassani et al. (2009) showed that the forecasting performance of parametric models are adversely affected by season dummies. In order to enable evaluating the sensitivity of the forecasting models to recessions, we divide the U.S. imports and exports series into three parts as follows, depending on the point at which the 2008 recession causes the major

[2]The optimal SSA code used in this study is available upon request.

structural break visible in the respective series. This enables us to analyse U.S. trade before, during and after the 2008 recession using the selected forecasting techniques.

- Imports: Before Great Recession (January 1989 - July 2008), During Great Recession (August2008 - April 2009) and After Great Recession (May 2009 - December 2011).
- Exports: Before Great Recession (January 1989 - June 2008), During Great Recession (July 2008 - January 2009) and After Great Recession (Feb 2009 - December 2011).

Initially the total U.S. imports and exports series are tested for structural breaks using the Bai and Perron (2003) test for breakpoints. The results from this test indicates that the total U.S. Imports series has four structural breaks corresponding to July-1994, May-1999, February-2004 and July-2007. The total U.S. Exports series too was reported to have four structural breaks corresponding to September-1993, February-1997, February-2004 and July-2007. In terms of determining the dates of regime switch, we have made use of the National Bureau of Economic Research (NBER) official U.S. recession dates[3], the Bai and Perron (2003) test for break points and logical reasoning based on a closer analysis of U.S. imports and exports time series. It is clear from Figure 1 that there are two easily visible break points in the U.S. trade series around 2001 and 2008 (corresponding to the two U.S. recessions) and that the break associated with the 2008 recession has a significantly higher magnitude in terms of its impact on trade in comparison to the 2001 recession. As we wish to evaluate the sensitiveness of the selected forecasting models before, during and after recessions, we decided to consider the break created by the most recent financial crisis as the cut-off point owing to its major impact with a comparatively higher magnitude. However, the Bai and Perron (2003) test suggests that both imports and exports in U.S. experienced a structural break in July 2007 well before the onset of the 2008 recession. As we know the official U.S. recession dates based on the NBER reports, we decided to take a closer look at each time series to determine the peaks and troughs in U.S. trade manually. Accordingly, based on NBER the U.S. economy peaked in December 2007 and experienced a trough in June 2009. However, when we analysed the U.S. trade data it was clear that the peak in imports and exports occurred in July 2008 and June 2008 respectively whilst the trough sets in during April 2009 and January 2009 respectively. As such we have selected these dates to represent the regime switch in this paper as it enables capturing the exact month during which the recession affected U.S. imports and exports series. The appropriateness of the approach used to select the regime switch dates are clear through the following example. Suppose we split the U.S. exports series in June 2007 based on the Bai and Perron (2003) recommendation, then the value corresponding to this point would be \$97,886 (millions). However, a closer look at the series shows that following this point in time, U.S. exports continues to rise further to peak at \$118,717 (millions) in June 2008 and thereafter experiences a rapid decline which is the major structural break visible to the naked eye in Figure 1.

For each scenario (except in the case of during the 2008 recession) we leave aside approximately $\frac{1}{3}^{rd}$ of the data to test the forecasting accuracy of the models and use approximately $\frac{2}{3}^{rd}$ of the observations for training each model. During the recession, owing to data constraints the analysis is limited to a horizon of one month ahead. For the remaining scenarios we analyse the data at horizons of $h = 1$, 3, 6 and 12 steps ahead enabling us to capture both short and long term fluctuations in U.S. trade.

Table 1 in Appendix C presents some descriptive statistics for U.S. imports and exports. The descriptive statistics show that during the period from January 1989 to December 2011,

[3]http://www.nber.org/cycles/cyclesmain.html.

total U.S. imports of goods averaged at US$ 99,230 whilst total exports averaged US$ 64,710. Accordingly it is evident that during this period the U.S. did indeed undergo continuous trade deficits as mentioned earlier. The minimum and maximum columns show that during the 2008 recession, U.S. imports declined by 47% whilst exports declined by 41%. Yet, the trade balance continued to remain in deficit over this period. Krugman (2008) states that historically, the U.S. traded heavily with the four Asian tigers and other developed nations, but in the recent past there has been a change in trading partners with the U.S. increasing its trade with Asian countries, China and Mexico. This in turn explains the reason for a lower decline in exports in comparison to the decline in imports during the 2008 recession. Many Asian countries, and China in particular were not as badly affected by the 2008 recession in comparison to the U.S. and European nations which were entangled in the sub-prime fiasco. Accordingly, China and Asian countries continued to demand for U.S. exports on a lower scale than usual.

Interestingly, the standard deviation (SD) of exports in U.S. is much lower than the SD of imports. This suggests that during the period in question, U.S. imports were more volatile in comparison to exports. However, it is not prudent to rely on the SD as a measure of variability in the case of U.S. trade because there exists a significant variation in the average value of each time series. As such the coefficient of variation (CV) is used to comment on the variability. Based on the CV, we can conclude that imports are in fact more variable than exports, and that same relationship holds between imports and exports, both before and during the 2008 recession. However after the recession, exports are seen to have more variation than imports post-recession. Furthermore, the skewness statistic (not reported here) showed that total U.S. trade data is positively skewed. This in turn meant that U.S. trade data does not follow a normal distribution and this was confirmed via normality testing using the Shapiro-Wilk (S-W) test with the same non-normality results being applicable to both imports and exports before the 2008 recession. The kurtosis (not reported here) analysis showed that U.S. imports and exports have a Platykurtic distribution, which means the data is distributed much flatter than a normal distribution, with a wider peak and with values more likely to be spread around the mean. However, both imports and exports data were found to be normally distributed based on the S-W test during and after the 2008 recession. Finally, we tested the data for a unit root problem using the Augmented Dickey-Fuller (ADF) test. The results indicated that all series considered in this study suffered from a unit root problem, which in turn means the time series are non-stationary. This was expected especially owing to the two structural breaks visible in U.S. Trade series (Figure 1). This further confirms the earlier claims we made suggesting that structural breaks result in making a time series non-stationary in mean and variance. In addition, these tests also confirms the appropriateness of the selected data set, and the necessity to evaluate the impact of structural breaks on U.S. trade forecasting models which currently rely on both parametric and nonparametric forecasting techniques.

4 Empirical Results

4.1 Metrics

For comparing the forecasting accuracy between ARIMA, ETS, Neural Networks (NN) and SSA trade models, the Root Mean Squared Error (RMSE) and the direction of change (DC) criterions are adopted in this paper. All outcomes relating to forecasting accuracy are tested further for statistical significance using the modified Diebold-Mariano test in Harvey et al. (1997) whilst the DC results are tested for statistical significance using a student's t-test. It should be noted that due to data limitations, the post-recession, $h = 12$ step ahead forecast is limited to one observation and as such is not significant.

Root Mean Squared Error (RMSE)

The RMSE statistic is a popular and standard quantitative technique for evaluating the forecasting accuracy between two models. It is also popular as one of the most frequently cited measures in forecasting literature (see, for example, Zhang et al. (1998); Hassani et al. (2009,2012); Hassani and Mahmoudvand, (2013)). Here, we mainly follow Altavilla and Grauwe (2010) in defining the RMSE.

$$\text{RMSE} = \left(\frac{1}{n} \sum_{i=1}^{N} e^2_{t+h+i} \right)^{1/2},$$

where, $e_{t+k} = \chi_{t+h} - \widehat{\chi}_{t+h}$ is the forecast error where $h \geq 1$, and $\widehat{\chi}_{t+b}$ represents the h-step-ahead forecast.

If RMSE_{SSA} denotes the RMSE for forecasts from SSA, and RMSE_{ARIMA} denotes the RMSE for forecasts from ARIMA, then the Ratio of the RMSE (RRMSE) can be calculated as $\frac{RMSE_{SSA}}{RMSE_{ARIMA}}$. If this ratio is less than 1, it means that SSA outperforms ARIMA by 1-$(\frac{RMSE_{SSA}}{RMSE_{ARIMA}})$ percent and vice versa.

Direction of Change (DC)

The DC criterion is a measure of the percentage of accurate direction of change predictions from a given forecasting model. The DC metric is an equally important measure as the RMSE because it is important that for example, when the actual series is illustrating an upwards trend, the forecast is able to predict an upward trend and vice versa. As noted in Altavilla and Grauwe (2010), a model is said to have a better DC prediction than a random walk if it records a DC greater than 50%. A detailed description of the DC metric can be found in Altavilla and Grauwe (2010). In brief,

$$\text{DC} = \left(\frac{1}{T} \sum_{t=1}^{T} \phi \Delta^e_{t+h} = \Delta_{t+h} \right),$$

where, Δ_{t+h} and Δ^e_{t+h} are the actual and predicted direction of change in U.S. Trade h steps ahead, and ϕ equals 1 if $\Delta_{t+h} = \Delta^e_{t+h}$ and 0 otherwise.

4.2 Results

In this section, the forecasting results for U.S. imports and exports are analysed separately as; before, during and after the 2008 recession. Tables' 2-4 (see, Appendix C) report the RMSE for out-of-sample forecasting results of U.S. imports and exports before, during and after the 2008 recession.

The following observations relating to U.S. imports can be inferred from Table 2. Firstly, it is clear that before and after the 2008 recession, SSA outperforms ARIMA in forecasting U.S. imports in both the short and long run. Before the recession, the RRMSE results at horizons of h = 1, 3 and 12 are found to be statistically significant and accordingly in the very short run SSA forecasts are 20% better than the ARIMA forecasts whilst at 12 months ahead, SSA provides a 4% better forecast in comparison to ARIMA. Even though 4% appears to be an insignificant figure, it is important to note that this value is statistically significant at a p-value of 0.05 and that we are evaluating figures in millions of U.S. Dollars. After the recession, the only statistically significant result between SSA and ARIMA is the one month ahead forecast whereby SSA is seen providing a forecast which is 11% better than the forecasts from the ARIMA model. Next, we consider the forecasting performance between SSA and ETS. The ETS model is able to provide a more accurate forecast than SSA for U.S. imports before the 2008 recession at $h = 12$ months ahead, but after the recession it appears that SSA is able to provide a more accurate forecast for U.S. imports in comparison to ETS at all horizons. We find statistically significant evidence to prove that the forecasts from SSA outperforms the forecasts from ETS by 22% at one month ahead for U.S. imports before the recession, whilst the ETS model provided forecasts which are 5% better than those from the SSA model at a horizon of 12 months ahead before the recession. After the recession we are able to conclude with 90% confidence that forecasts from the SSA model outperforms forecasts via ETS by 24% at $h = 1$ month ahead. When comparing the NN and SSA models we find that before the recession SSA's forecasts outperforms forecasts from the NN model at all horizons with statistically significant results, but after the recession at $h = 12$ steps ahead the NN forecast was able to outperform the SSA forecast at forecasting U.S. imports. However as noted earlier this value is based on one observation alone and the result is not statistically significant. Furthermore, post-recession, we find statistically significant evidence to conclude that the SSA model's forecast is 44% better than the NN model forecast for U.S. imports at $h = 6$ months ahead.

Next, we analyse the average RRMSE columns for U.S. imports. The first observation is that before the 2008 recession, forecasts from SSA are on average 11% better than forecasts from ARIMA, 9% better than forecasts from ETS and 51% better than forecasts from the NN model. However, following the recession we find that on average forecasts from SSA are 34% better than forecasts from ARIMA and 42% better than forecasts from ETS. It is clear that the forecasting accuracy of the two parametric models have deteriorated significantly post-recession. The SSA model's forecasting accuracy increases significantly in comparison to the two parametric models following the 2008 recession. In contrast, the NN model appears to provide a better fit than it did prior to the 2008 recession, and is in fact the only other model (in addition to SSA) which reports an increase in average forecasting accuracy following the structural break. However, forecasts from SSA are on average found to be 27% better than those from the NN model when forecasting U.S. imports post-recession. The initial results suggest that for U.S. imports forecasting, the parametric models are highly sensitive to the structural break caused by the 2008 recession and that the nonparametric models perform better and appear to be less sensitive to structural breaks. The results further indicate that before and after a recession, SSA is able to provide the best forecast for U.S. imports outperforming forecasts from the optimal ARIMA model, ETS and NN. Whilst it is arguable that the post-recession sample size hinders modelling capabilities, these results indicate the SSA model is able to handle small sample sizes comparatively better than ARIMA and ETS. The SSA technique only requires a minimum of three observations in order to provide an accurate forecast, and this feature is exploited when forecasting U.S. trade during the recession.

Table 3 reports the out-of-sample forecasting results for U.S. exports. Before the recession, based on the RMSE, SSA's forecasts outperforms ARIMA and ETS at forecasting U.S. exports at horizons of $h = 1$ and 3 steps ahead, whilst forecasts from both ARIMA and ETS outperforms forecasts from SSA at horizons of $h = 6$ and 12 steps ahead. However, is it interesting to note that the results are only statistically significant at $h = 1$ and 3 steps ahead, whereby the RRMSE shows that the SSA forecasts are 43% and 30% better than ARIMA at forecasting U.S. imports, and that SSA is 43% and 37% better forecasts than ETS at the same horizons. Accordingly the positive long run forecasting results attained by ARIMA and ETS are likely to be chance occurrences and as such we conclude that there is no difference between the forecasts from of SSA, ARIMA and ETS in the long run in terms of forecast accuracy. According to Table 3, the SSA model outperforms NN at all horizons before the 2008 recession with statistically significant results. After the recession, SSA's forecasts outperform those from ARIMA, ETS and NN at h= 1, 3, and 6 steps ahead whilst the forecasts from ARIMA, ETS and NN are seen outperforming the SSA forecast at $h = 12$ steps ahead, however with statistically insignificant results as before. Following the 2008 recession we are able to conclude with 95% confidence that SSA is 48% better than ARIMA at providing $h = 3$ steps ahead forecasts for U.S. exports, and that forecasts from the SSA model is 11% better than ETS at forecasting U.S. exports at 6 months ahead.

We then analyse the average RRMSE values for U.S. exports. The results show that on average, before the 2008 recession, forecasts from SSA are 15% better than forecasts from ARIMA, 17% better than ETS and 74% better than NN. After the 2008 recession the SSA model's forecasts are on average 22% better than ARIMA at forecasting U.S. exports and 18% better than the NN model forecasts. However, following the recession, on average SSA forecasts were found to be only 0.002% more accurate than forecasts from the ETS model. This was largely due to the positive $h = 12$ months ahead result attained by ETS post-recession which was a chance occurrence relating to a single forecasting point. As such we compute the post recession average RRMSE values based on $h = 1$, 3, and 6 months ahead. The results then indicate that following the recession, SSA based forecasts are 31% better than ARIMA, 9% better than ETS and 33% better than those from the NN model. These results indicate that SSA continues to remain the least affected by the structural break, but it is interesting to see the ETS model's performance at U.S. exports forecasting improving following the 2008 recession. This could partly be attributed to chance occurrences as the SSA to ETS RRMSE results are only statistically significant at $h = 6$ steps ahead.

In order to ensure the robustness of the reported results, and also as we have included the 2001 structural break in the results before the great recession reported above, we considered the forecasting accuracy of models before the great recession by considering data from January 2002 - June 2008. The results are reported in Table 4. These results indicate that forecasts from the SSA technique continues to outperform forecasts from ARIMA, ETS and NN models for U.S. trade forecasting even in the absence of the break created by the 2001 recession. These results further confirm the robustness of the SSA model in comparison to the other forecasting techniques evaluated in this paper.

Next we evaluated using SSA, ARIMA, ETS and NN models for forecasting U.S. imports and exports during the 2008 recession. We found that ETS and NN models were not able to forecast U.S. imports and exports during the 2008 recession given the small sample size available for training the model. Accordingly, ETS was not able to fit a model and the *nnetar* function was not able to determine the weights. This further outlines the superiority of the SSA technique over ETS and NN as it is able to model accurately with small sample sizes and the minimum required sample in the case of SSA is 3 observations. In order to make the comparison more meaningful, we incorporate Holt-Winters as a forecasting model for during the recession. The results are reported through Table 5 in Appendix C. For the SSA model, we use a small trajectory matrix as adopted in Hassani et al. (2013b) and the results show this method enables SSA to provide a very good short horizon forecast. Accordingly, we find that during the 2008 recession, the SSA forecast is able to outperform both ARIMA and Holt-Winters (HW) forecasts for U.S. imports. The RRMSE shows that the SSA forecast is 40% better than ARIMA, and 6% better than HW's at predicting U.S. imports during the recession. However, it should be noted that these results are not statistically significant. For forecasting exports during the recession, SSA was once again able to outperform both ARIMA and Holt-Winters forecasts based on the RMSE. Interestingly, for U.S. exports forecasting, the statistically significant RRMSE makes it evident that the SSA forecast is 42% better than ARIMA, and 20% better than HW's at predicting U.S. exports.

Finally, we adopt the DC criterion to determine the percentage of forecasts that correctly identified the direction of change in U.S. imports and exports. The results are shown in Appendix C via Table 6. It is evident from the results that both before and after the 2008 recession, forecasts from SSA outperforms ARIMA and NN forecasts based on DC at all horizons for U.S. imports forecasting. For U.S. exports forecasting, SSA forecasts outperform ARIMA and ETS at all horizons both before and after the recession, with the exception of the $h = 6$ steps ahead forecast before the 2008 recession. However, given that SSA reports a higher DC value in comparison to ARIMA at $h = 12$, and that the earlier results based on RMSE and RRMSE showed that when ARIMA outperformed SSA at forecasting exports the results were not statistically significant, it is possible to conclude that SSA is more suitable for U.S. exports forecasting before and after the 2008 recession in comparison to ARIMA. As with imports forecasting, for U.S. exports too the SSA outperformed NN at all horizons before and after the recession based on the DC criterion. The DC results which covers the period during the recession are not reported here, as once again SSA was seen outperforming both ARIMA and Holt-Winters at correctly predicting the direction of change in U.S. imports and exports during the 2008 recession.

5 Conclusion

In this paper, we have introduced SSA for trade forecasting and compared the forecasting accuracy of SSA against the optimal ARIMA model, ETS, and NN models before, during and after the 2008 recession. Our results indicate that following a recession, forecasts from SSA can outperform ARIMA, ETS and NN at predicting U.S. imports and exports based on the RMSE, RRMSE and the DC criterions along with statistically significant results in most cases. Accordingly we have shown here that following a major structural break such as the one caused by the onset of the 2008 recession, the forecasting ability of ARIMA, and ETS models are adversely affected whilst the impact on the nonparametric NN model is worse than the effect on the SSA model. The results also show that in comparison to ARIMA, ETS and NN, the SSA model is less sensitive to a structural break and is therefore able to model and forecast more accurately following a recession. During a recession the results show that forecasts from SSA can provide the most accurate prediction for U.S. imports and exports in comparison to both ARIMA and Holt-Winters whilst ETS and NN fails to train models when faced with small sample sizes. The impressive performance of the SSA model can be attributed to its nonparametric nature and sound noise filtering capabilities.

In terms of future research, this paper opens up two clear avenues. Firstly, our findings indicate the possibility of attaining further enhanced forecasts by adopting a tailored SSA model for forecasting total U.S. imports and exports. This would be via the incorporation of the Vector SSA model with change point detection for automatically detecting structural breaks in the time series and switching to a small trajectory matrix approach for forecasting during a recession or an expansion, and thereafter reverting back to its original state. When adopting SSA combined with change point detection for U.S. trade forecasting in the future, it would be pertinent to consider Markov switching, smooth transition and time varying specifications for comparison purposes against SSA. Secondly, future research could ascertain whether a hybrid model combining SSA and ARIMA could help outperform the SSA forecast and improve ARIMA's current forecasting accuracy. Such an approach could consider using the SSA technique for filtering the trade series and the ARIMA model for obtaining forecasts.

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Appendix

A ETS Formula

Trend	Seasonal N	Seasonal A	Seasonal M
N	$\hat{y}_{t+h\|t} = \ell_t$ $\ell_t = \alpha y_t + (1-\alpha)\ell_{t-1}$	$\hat{y}_{t+h\|t} = \ell_t + s_{t-m+h_m^+}$ $\ell_t = \alpha(y_t - s_{t-m}) + (1-\alpha)\ell_{t-1}$ $s_t = \gamma(y_t - \ell_{t-1}) + (1-\gamma)s_{t-m}$	$\hat{y}_{t+h\|t} = \ell_t s_{t-m+h_m^+}$ $\ell_t = \alpha(y_t / s_{t-m}) + (1-\alpha)\ell_{t-1}$ $s_t = \gamma(y_t / \ell_{t-1}) + (1-\gamma)s_{t-m}$
A	$\hat{y}_{t+h\|t} = \ell_t + hb_t$ $\ell_t = \alpha y_t + (1-\alpha)(\ell_{t-1} + b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1}$	$\hat{y}_{t+h\|t} = \ell_t + hb_t + s_{t-m+h_m^+}$ $\ell_t = \alpha(y_t - s_{t-m}) + (1-\alpha)(\ell_{t-1} + b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1}$ $s_t = \gamma(y_t - \ell_{t-1} - b_{t-1}) + (1-\gamma)s_{t-m}$	$\hat{y}_{t+h\|t} = (\ell_t + hb_t)s_{t-m+h_m^+}$ $\ell_t = \alpha(y_t / s_{t-m}) + (1-\alpha)(\ell_{t-1} + b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1}$ $s_t = \gamma(y_t / (\ell_{t-1} + b_{t-1})) + (1-\gamma)s_{t-m}$
$\mathbf{A_d}$	$\hat{y}_{t+h\|t} = \ell_t + \phi_h b_t$ $\ell_t = \alpha y_t + (1-\alpha)(\ell_{t-1} + \phi b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)\phi b_{t-1}$	$\hat{y}_{t+h\|t} = \ell_t + \phi_h b_t + s_{t-m+h_m^+}$ $\ell_t = \alpha(y_t - s_{t-m}) + (1-\alpha)(\ell_{t-1} + \phi b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)\phi b_{t-1}$ $s_t = \gamma(y_t - \ell_{t-1} - \phi b_{t-1}) + (1-\gamma)s_{t-m}$	$\hat{y}_{t+h\|t} = (\ell_t + \phi_h b_t)s_{t-m+h_m^+}$ $\ell_t = \alpha(y_t / s_{t-m}) + (1-\alpha)(\ell_{t-1} + \phi b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)\phi b_{t-1}$ $s_t = \gamma(y_t / (\ell_{t-1} + \phi b_{t-1})) + (1-\gamma)s_{t-m}$
M	$\hat{y}_{t+h\|t} = \ell_t b_t^h$ $\ell_t = \alpha y_t + (1-\alpha)\ell_{t-1}b_{t-1}$ $b_t = \beta^*(\ell_t / \ell_{t-1}) + (1-\beta^*)b_{t-1}$	$\hat{y}_{t+h\|t} = \ell_t b_t^h + s_{t-m+h_m^+}$ $\ell_t = \alpha(y_t - s_{t-m}) + (1-\alpha)\ell_{t-1}b_{t-1}$ $b_t = \beta^*(\ell_t / \ell_{t-1}) + (1-\beta^*)b_{t-1}$ $s_t = \gamma(y_t - \ell_{t-1}b_{t-1}) + (1-\gamma)s_{t-m}$	$\hat{y}_{t+h\|t} = \ell_t b_t^h s_{t-m+h_m^+}$ $\ell_t = \alpha(y_t / s_{t-m}) + (1-\alpha)\ell_{t-1}b_{t-1}$ $b_t = \beta^*(\ell_t / \ell_{t-1}) + (1-\beta^*)b_{t-1}$ $s_t = \gamma(y_t / (\ell_{t-1}b_{t-1})) + (1-\gamma)s_{t-m}$
$\mathbf{M_d}$	$\hat{y}_{t+h\|t} = \ell_t b_t^{\phi_h}$ $\ell_t = \alpha y_t + (1-\alpha)\ell_{t-1}b_{t-1}^{\phi}$ $b_t = \beta^*(\ell_t / \ell_{t-1}) + (1-\beta^*)b_{t-1}^{\phi}$	$\hat{y}_{t+h\|t} = \ell_t b_t^{\phi_h} + s_{t-m+h_m^+}$ $\ell_t = \alpha(y_t - s_{t-m}) + (1-\alpha)\ell_{t-1}b_{t-1}^{\phi}$ $b_t = \beta^*(\ell_t / \ell_{t-1}) + (1-\beta^*)b_{t-1}^{\phi}$ $s_t = \gamma(y_t - \ell_{t-1}b_{t-1}^{\phi}) + (1-\gamma)s_{t-m}$	$\hat{y}_{t+h\|t} = \ell_t b_t^{\phi_h} s_{t-m+h_m^+}$ $\ell_t = \alpha(y_t / s_{t-m}) + (1-\alpha)\ell_{t-1}b_{t-1}^{\phi}$ $b_t = \beta^*(\ell_t / \ell_{t-1}) + (1-\beta^*)b_{t-1}^{\phi}$ $s_t = \gamma(y_t / (\ell_{t-1}b_{t-1}^{\phi})) + (1-\gamma)s_{t-m}$

Figure 2: Formulae for recursive calculations and point forecasts (Hyndman and Athanasopoulos, 2013).

B Optimal VSSA Forecasting Algorithm

1. Consider a real-valued nonzero time series $Y_N = (y_1, \ldots, y_N)$ of length N.

2. Divide the time series into two parts; $\frac{2}{3}^{rd}$ of observations for model training and testing, and $\frac{1}{3}^{rd}$ for validating the forecast accuracy.

3. Use the training data to construct the trajectory matrix $\mathbf{X} = (x_{ij})_{i,j=1}^{L,K} = [X_1, ..., X_K]$, where $X_j = (y_j, ..., y_{L+j-1})^T$ and $K = N - L + 1$. Beginning with $L = 2$ $(2 \leq L \leq \frac{N}{2})$, evaluate all possible values of L for Y_N.

4. Obtain the SVD of $\mathbf{X}$ by calculating $\mathbf{XX}^T$ for which $\lambda_1, \ldots, \lambda_L$ denotes the eigenvalues in decreasing order $(\lambda_1 \geq \ldots \lambda_L \geq 0)$ and by $U_1, \ldots, U_L$ the corresponding eigenvectors. The resulting output at this stage is $\mathbf{X} = \mathbf{X}_1 + \ldots + \mathbf{X}_L$ where $\mathbf{X}_i = \sqrt{\lambda_i} U_i V_i^T$ and $V_i = \mathbf{X}^T U_i / \sqrt{\lambda_i}$.

5. Evaluate all possible combinations of r $(1 \leq r \leq L-1)$ singular values (step by step) for the selected L and split the elementary matrices $\mathbf{X}_i$ $(i = 1, \ldots, L)$ into several groups and sum the matrices within each group.

6. Perform diagonal averaging to transform the matrix with the selected r singular values into a Hankel matrix which can then be converted into a time series (the steps up to this stage filters the noisy series). The output is a filtered series that can be used for forecasting.

7. Set $v^2 = \pi_1^2 + \ldots + \pi_r^2$, where π_i represents the final component of the eigenvector U_i $(i = 1, \ldots, r)$. Assume that, $e_L = (0, 0...., 1)$ is not a component of the linear space $\mathfrak{L}_r$, which implies $\mathfrak{L}_r$ is not a vertical space.

8. Consider the matrix $\Pi = \mathbf{V}^\triangledown(\mathbf{V}^\triangledown)^T + (1-v^2)AA^T$, where $A = (a_1, \ldots, a_{L-1}) = \sum_{i=1}^r \pi_i U_i^\triangledown/(1-v^2)$ and $\mathbf{V}^\triangledown = [U_1^\triangledown, ..., U_r^\triangledown]$, where $\mathbf{V}^\triangledown$ is the first $L-1$ components.

9. Next, consider the linear operator $\theta^{(v)} : \mathfrak{L}_r \mapsto \mathbf{R}^L$, where $\theta^{(v)}U = \begin{pmatrix} \Pi U^\triangledown \\ A^T U^\triangledown \end{pmatrix}$.

10. Then, define vector Z_i after grouping and eliminating noise components, such that

$$Z_i = \begin{cases} \widetilde{X}_i & \text{for } i = 1, \ldots, K \\ \theta^{(v)} Z_{i-1} & \text{for } i = K+1, \ldots, K+h+L-1, \end{cases}$$

where, $\widetilde{X}_i$'s are the reconstructed columns of the trajectory matrix.

11. Construct the matrix $\mathbf{Z} = [Z_1, ..., Z_{K+h+L-1}]$ and perform diagonal averaging to obtain a new series $y_1, ..., y_{N+h+L-1}$, where $y_{N+1}, ..., y_{N+h}$ forms the h terms of the SSA Vector forecast.

12. Define a loss function $\mathcal{L}$.

13. When forecasting a series Y_N h-step ahead, the forecast error is minimised by setting $\mathcal{L}(X_{K+h} - \hat{X}_{K+h})$ where the vector $\hat{X}_{K+h}$ contains the h-step ahead forecasts obtained using the VSSA forecasting algorithm.

14. Find the combination of L and r which minimises $\mathcal{L}$ and thus represents the optimal VSSA choices.

15. Use the optimal VSSA choices to obtain the forecast for the validation set.

Beyond its simplicity, the popularity of forecasting models such as ARIMA stems from its in-sample and dynamic forecasting performance. Accordingly, prior to introducing the VSSA forecasting algorithm, we find it pertinent to briefly comment on the similarities between ARIMA and SSA. In doing so, we mainly follow Hassani and Thomakos (2010) which provides a detailed account. For example, if we denote β as a fixed $(L \times 1)$ vector, then when $\beta = [-1, 1]$ and $L = 2$ we have the first differences of the realization as $\beta\mathbf{X}$. Furthermore, setting $L \geq 2$ and $\beta = [1/L, 1/L, \ldots, L]$ gives us a L-order moving average for the realization as $\beta\mathbf{X}$. Moreover, the linear recurrent formula which is used for forecasting in SSA is

$$y_{i+d} = \sum_{k=1}^{d} \alpha_k y_{i+d-k}, \tag{7}$$

where $1 \leq i \leq N - d$, is closely identical in structure to autoregressive models even though the calculation of the parameters differ.

C Tables

Table 1: Descriptive Statistics for U.S. Imports and Exports (1989-2011).

Series	Mean	Med.	Min.	Max.	SD	CV	S-W(p)	ADF
Imports								
Total	99230	92880	36440	205900	47440	48	0.00	1.40†
Before Great Recession	88460	78700	36440	205900	41815	47	0.00	3.20†
During Great Recession	147800	139300	110100	192800	32498	22	0.11*	-1.67†
After Great Recession	164700	168800	117400	198800	22623	14	0.17*	1.67†
Exports								
Total	64710	59040	26160	133400	26102	40	0.00	1.69†
Before Great Recession	57100	56580	26160	118700	19461	34	0.00	2.32†
During Great Recession	103200	108100	79280	118000	14656	14	0.41*	-2.28†
After Great Recession	107900	108100	81410	133400	15902	15	0.09*	1.51†

Note:* indicates a normal distribution based on a Shapiro-Wilk test at $p = 0.01$.
† indicates non-stationarity based on an Augmented Dickey-Fuller test at $p = 0.01$.

Table 2: U.S. Imports out-of-sample forecasting results.

h	ARIMA	ETS	NN	SSA	$\frac{SSA}{ARIMA}$	$\frac{SSA}{ETS}$	$\frac{SSA}{NN}$
Before Great Recession							
1	6929	7143	15402	5538	0.80*	0.78*	0.36*
3	9348	9264	12636	7679	0.82*	0.83	0.61*
6	8926	8790	15604	8622	0.97	0.98	0.55*
12	9711	8880	19536	9319	0.96**	1.05**	0.48*
Average	8729	8519	15797	7790	0.89	0.91	0.49
After Great Recession							
1	10380	12122	11009	9269	0.89*	0.76**	0.84
3	17008	16286	17695	7419	0.44	0.46	0.42
6	8307	9449	11960	7930	0.95	0.84	0.66*
12	10387	14316	937	5784	0.70	0.40	6.17
Average	11520	13043	10400	7600	0.66	0.58	0.73

Note:* indicates results are statistically significant based on Diebold-Mariano at $p = 0.05$.
** indicates statistical significance at $p = 0.10$.

Table 3: U.S. Exports out-of-sample forecasting results.

h	ARIMA	ETS	NN	SSA	$\frac{SSA}{ARIMA}$	$\frac{SSA}{ETS}$	$\frac{SSA}{NN}$
Before Great Recession							
1	4321	4407	12509	2480	0.57*	0.56*	0.20*
3	4558	5100	14257	3198	0.70*	0.63*	0.22*
6	4701	4757	15515	4834	1.03	1.02	0.31*
12	6234	6050	21661	6337	1.02	1.05	0.29*
Average	4953	5079	15986	4213	0.85	0.83	0.26
After Great Recession							
1	7951	5925	7681	5776	0.73	0.97	0.75
3	10539	6284	10225	5441	0.52*	0.87	0.53
6	7542	7106	8534	6325	0.84	0.89*	0.74
12	3735	4225	2307	5943	1.59	1.41	2.58
Average	7442	5885	7187	5871	0.78	0.998	0.82

Note:* indicates results are statistically significant based on Diebold-Mariano at $p = 0.05$.

Table 4: U.S. Trade out-of-sample forecasting results.

h	ARIMA	ETS	NN	SSA	$\frac{SSA}{ARIMA}$	$\frac{SSA}{ETS}$	$\frac{SSA}{NN}$
Imports: Before Great Recession (Data: January 2002 - July 2008)							
1	9184	8701	14879	6958	0.76*	0.80*	0.47*
3	11464	11180	12896	10488	0.91	0.94	0.81*
6	10582	10998	14508	10501	0.99	0.72*	0.72*
12	9672	8717	19565	9236	0.95*	1.06*	0.47*
Average	10226	9899	15462	9296	0.91	0.94	0.60
Exports: Before Great Recession (Data: January 2002 - June 2008)							
1	5331	5085	7420	3333	0.63*	0.66*	0.45*
3	5740	5704	7161	3759	0.65*	0.66*	0.52*
6	3747	4791	11239	3913	1.04	0.82*	0.35*
12	4695	6346	15560	4709	1.00	0.74*	0.30*
Average	4878	5482	10345	3929	0.81	0.72	0.38

Note:* indicates results are statistically significant based on Diebold-Mariano at $p = 0.10$.

Table 5: During recession: Imports and Exports forecasting results.

h	ARIMA	HW	SSA	$\frac{SSA}{ARIMA}$	$\frac{SSA}{HW}$
Imports					
1	30157	19231	18045	0.60	0.94
Exports					
1	12683	9241	7418	0.58*	0.80*

Note:* indicates results are statistically significant based on Diebold-Mariano at $p = 0.05$.

Table 6: Direction of change results for U.S. Imports and Exports.

h	ARIMA	ETS	NN	SSA
Imports				
Before Great Recession				
1	0.57	0.63	0.59	0.78
3	0.57	0.68	0.52	0.82*
6	0.76	0.73	0.57	0.80
12	0.96*	1.00*	0.44	1.00*
After Great Recession				
1	0.42	0.67	0.58	0.50
3	0.50	0.50	0.50	0.70
6	0.57	0.86	0.57	0.86
12	1.00	1.00	1.00	1.00
Exports				
Before Great Recession				
1	0.67	0.73	0.50	0.79*
3	0.79*	0.67	0.57	0.87*
6	0.84*	0.82	0.51	0.75
12	0.87	0.90	0.54	0.93**
After Great Recession				
1	0.50	0.67	0.58	0.92**
3	0.50	0.70	0.60	0.70
6	0.86	0.86	0.71	1.00*
12	1.00	1.00	1.00	1.00

Note:* indicates results are statistically significant based on a t test at $p = 0.05$.